

UB-ECM-PF 95/2
February 1995

On black holes in the theory of dilatonic gravity coupled to a scalar field

E. Elizalde^{a,b,1}, **P. Fosalba-Vela**^a, **S. Naftulin**^c and **S.D. Odintsov**^{b,2}

^aCenter for Advanced Study CEAB, CSIC, Camí de Sta. Bárbara, 17300 Blanes

^bDepartment ECM and IFAE, Faculty of Physics, University of Barcelona,
Diagonal 647, 08028 Barcelona, Catalonia, Spain

^cInstitute for Single Crystals, 60 Lenin Ave., 310141 Kharkov, Ukraine

Abstract

Taking advantage of the representation of dilatonic gravity with the R^2 -term under the form of low-derivative dilatonic gravity coupled to an additional scalar, we construct a general renormalizable model motivated by this theory. Exact black hole solutions are found for some specific versions of the model, and their thermodynamical properties are described in detail. In particular, their horizons and temperatures are calculated. Finally, the corresponding one-loop effective action is obtained in the conformal gauge, and a number of its properties—including the construction of one-loop finite models—are briefly described.

PACS: 04.62.+v, 04.60.-m, 02.30.+g

¹E-mail: eli@zeta.ecm.ub.es

²On leave of absence from Tomsk Pedagogical Institute, 634041 Tomsk, Russia. E-mail: odintsov@ecm.ub.es

1. Introduction. Recently, the study of two-dimensional (2D) dilatonic gravity has become very interesting, both at the classical and at the quantum level, for a variety of reasons. In the first place, this theory is closely connected with string theory (see [1] for a review on this point), where it appears as a sort of effective action. Secondly, dilatonic gravity itself can be represented under the form of a sigma model (see, for example, Ref. [2]), what helps to understand the connection between the conformal properties of the corresponding sigma model and the solutions of dilatonic gravity [3]. Finally, the hope exists that through the study of ‘easier’ 2D models one can get some insights useful for the investigation of realistic 4D gravity.

So far, the main activity has been concentrated on the study of ordinary dilatonic gravity with matter. However, there are different motivations for studying in such theory —in addition to the dilaton— another scalar of similar nature. For example, in string theory this scalar is the modulus field, which is connected with the radius of the compactified space (see, for example, [4]). In charged string theory there also appears an additional scalar, called spectator field (see, for instance, [5]). The additional scalar may be interpreted as a Liouville field [6], in some cases. Furthermore, as we will show in the next section, the scalar field appears in dilatonic gravity with an R^2 -term, on lowering the number of derivatives.

In this letter we formulate the theory of renormalizable dilatonic gravity with matter, coupled to a scalar field (Sect. 2). The study of solutions of black hole type for a few different variants of such theory is carried out in Sect. 3, with a systematic description of their properties. Finally, Sect. 4 is devoted to the study of the one-loop effective action of the model under consideration.

2. Dilatonic gravity coupled to a scalar field. We start from the theory of dilatonic gravity with an R^2 -term. The Lagrangian can be written in the following form

$$L = \frac{1}{2}Z(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + C(\phi)R + V(\phi) + \omega(\phi)R^2 - \frac{1}{2}f(\phi)g^{\mu\nu}\partial_\mu\chi_i\partial_\nu\chi^i, \quad (1)$$

where ϕ is the dilaton field, χ_i are scalars ($i = 1, 2, \dots, n$), and where the dilatonic functions $Z(\phi)$, $C(\phi)$, $V(\phi)$, $\omega(\phi)$ and $f(\phi)$ are assumed to be analytic in ϕ . At the classical level, the theory (1) represents a particular case of the most general theory of higher-derivative dilatonic gravity that was studied in Ref. [7]. The theory is motivated by the consideration of string theory in the background of massive modes (for a recent discussion see, for instance, [8]). At the quantum level, however, the theory (1) belongs to a different class than the models of [7]. In particular, one is not able to obtain the one-loop effective action of theory (1) from the general one-loop effective action in [7] (since the corresponding limit is singular).

As is well known, the calculations in the theory of higher-derivative quantum gravity are very involved (for a review, see [9]) and it is always preferable to work, when possible, with a low-derivative theory. One can reduce the order of the derivatives in (1) by introducing an auxiliary scalar, $\psi = 2\omega(\phi)R$, according to the trick in Ref. [10]:

$$\begin{aligned} S = & \int d^2x \sqrt{-g} \left\{ \frac{1}{2}Z(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + [C(\phi) + \psi]R - \frac{1}{4\omega(\psi)}\psi^2 + V(\phi) \right. \\ & \left. - \frac{1}{2}f(\phi)g^{\mu\nu}\partial_\mu\chi_i\partial_\nu\chi^i \right\}. \end{aligned} \quad (2)$$

This new theory is equivalent to the theory (1). At the same time, we get in this way an example of dilatonic gravity coupled with an additional scalar, which can be also interpreted

as a Liouville field or a matter scalar, or as a kind of spectator field —as we have mentioned in the introduction.

For what has been said, it is interesting to spend some time in the construction of the most general theory of renormalizable dilatonic gravity coupled to an additional scalar field. Starting from (1), its natural generalization looks like

$$S = \int d^2x \sqrt{-g} \left\{ \frac{1}{2} Z_{(ij)}(\phi) g^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi^j + C(\phi) R + V(\phi) - \frac{1}{2} f(\phi) g^{\mu\nu} \partial_\mu \chi_a \partial_\nu \chi^a \right\}, \quad (3)$$

where we have now two scalars ϕ_i , $i = 1, 2$, representing the dilaton and Liouville (or matter) scalar field, while the χ_a are the matter scalars. The kinetic matrix $Z_{(ij)}(\phi)$ is considered to be symmetric and the functions $C(\phi)$, $V(\phi)$ and $f(\phi)$ depend now (obviously) on both fields ϕ_1 and ϕ_2 . It is easy to see that the theory (3) is renormalizable in a generalized sense (unlike Einstein's 2D gravity [11]). Its one-loop renormalizability will be discussed in Sect. 4.

3. Solutions of black hole type. To begin with, we will consider some simple versions of the theory (1) (without matter), at the classical level, and will search for solutions of black hole type. In all we will consider four different models, each of them being a particular representative of (3), given by the Lagrangians:

$$L_I = \frac{1}{8\pi G} \psi(R + \Lambda_1) + bg^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \gamma \phi R + e^{-2a\phi} \Lambda, \quad (4)$$

$$L_{II} = \frac{1}{8\pi G} \left(\psi R - \frac{\psi^2}{4\omega(\phi)} \right) + bg^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \gamma \phi R + e^{-2a\phi} \Lambda, \quad (5)$$

$$L_{III} = \frac{1}{8\pi G} \psi(R + \Lambda_1) + e^{a_1\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + e^{-2a\phi} \Lambda + \gamma e^{a_2\phi} R, \quad (6)$$

$$L_{IV} = \frac{1}{8\pi G} \left(\psi R - \frac{\psi^2}{4\omega(\phi)} \right) + e^{a_1\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + e^{-2a\phi} \Lambda + \gamma e^{a_2\phi} R. \quad (7)$$

The first Lagrangian, L_I , describes Jackiw-Teitelboim (JT) dilatonic gravity [12], with the dilaton ψ interacting with a Liouville theory (ϕ is the Liouville field). Model II corresponds to the interaction of some different dilatonic gravity with a Liouville theory. This model can be also interpreted (after eliminating ψ and rescaling $\gamma \rightarrow \gamma/(8\pi G)$) as a particular version of dilaton gravity with an R^2 -term (1). R.B. Mann has recently considered black hole type solutions in models similar to (4) and (5) (albeit with a different dilatonic gravity part, see [6, 13]). Here we will use a technique similar to that of Ref. [6], specially to calculate the ADM mass (in the limit $x \rightarrow \infty$). Model III represents the interaction of JT gravity with a bosonic string-like effective action. Finally, the Lagrangian IV describes another version of the dilatonic gravity (1) with an R^2 -term (again with $\gamma \rightarrow \gamma/(8\pi G)$).

Choosing a static metric of the form

$$ds^2 = -g(x)dt^2 + g(x)^{-1}dx^2 \quad (8)$$

one can solve the classical field equations and try to find configurations corresponding to black holes. The results are given in Tables 1 to 4. Two general comments are in order. Concerning the \pm sign of the horizon, it is understood that when there is only one horizon

Characteristics	Model I
Fields	$g(x) = \frac{a^2 \Lambda}{b} + Dx + \frac{\Lambda_1}{2} x^2, \quad \psi(x) = Bx, \quad \phi(x) = \frac{1}{a} \ln x$
Parameters	$D = \frac{4\pi Gb}{a^2 B} \Lambda_1, \quad B \neq 0$ arbitrary
M (ADM mass)	$-\frac{2\pi Gb^2}{a^4 B} \Lambda_1 + \frac{Ba^2}{8\pi Gb} \Lambda$
BH criteria	(i) $\Lambda_1 > 0 \quad (B < 0 \text{ always})$
two horizons	if $\Lambda > 0, \quad b > 0, \quad b^3 > \frac{a^6 B^2 \Lambda}{8\pi^2 G^2 \Lambda_1}$
one horizon	if $\Lambda \cdot b < 0$
BH criteria	(ii) $\Lambda_1 < 0 \quad (B > 0 \text{ always})$
two horizons	if $\Lambda > 0, \quad b < 0 \quad b^3 > \frac{a^6 B^2 \Lambda}{8\pi^2 G^2 \Lambda_1 }$
one horizon	if $\Lambda \cdot b > 0$
Curvature	$-\Lambda_1$
Horizon	$-\frac{4\pi Gb}{a^2 B} \pm \sqrt{\left(\frac{4\pi Gb}{a^2 B}\right)^2 - \frac{2a^2 \Lambda}{\Lambda_1 B}}$
Temperature	$\frac{ \Lambda_1 }{4\pi} \sqrt{\left(\frac{4\pi Gb}{a^2 B}\right)^2 - \frac{2a^2 \Lambda}{\Lambda_1 B}}$

Table 1: Characteristics of the solutions of black hole type corresponding to model I.

Characteristics	Model II
Fields	$g(x) = \frac{a^2 \Lambda}{b} + Ax + Cx^2, \quad \psi(x) = -\frac{8\pi Gb}{a^2}, \quad \phi(x) = \frac{1}{a} \ln x$
Parameters	$C = \frac{2\pi Gb}{a^2 \omega_0}, \quad A \neq 0$ arbitrary, $\omega(\phi) = \omega_0 = \text{const.}$
M (ADM mass)	$-\frac{bA}{2a^2}$
BH criteria	(i) $b/\omega_0 > 0$
two horizons	if $\Lambda > 0, \quad A < 0, \quad A^2 > \frac{8\pi G\Lambda}{\omega_0}$
one horizon	if $A \cdot \Lambda > 0$
BH criteria	(ii) $b/\omega_0 < 0$
two horizons	if $\Lambda < 0, \quad A < 0, \quad A^2 > \frac{8\pi G\Lambda}{\omega_0}$
one horizon	if $A \cdot \Lambda < 0$
Curvature	$-\frac{4\pi Gb}{a^2 \omega_0}$
Horizon	$\frac{\omega_0 A }{4\pi Gb} \left(-1 \pm \sqrt{1 - \frac{8\pi G\Lambda}{\omega_0 A^2}} \right)$
Temperature	$\frac{ A }{4\pi} \sqrt{1 - \frac{8\pi G\Lambda}{\omega_0 A^2}}$

Table 2: Characteristics of the solutions of black hole type corresponding to model II.

Characteristics	Model III
Fields	$g(x) = C + Dx + \frac{\Lambda_1}{2}x^2, \psi(x) = \frac{A}{x^2} + \frac{B}{x^4}, \phi(x) = E \ln x$
Parameters	$a_1 = \frac{a_2}{2} = -\frac{4a}{3}, A = \frac{64\pi G}{9a_2^2} \frac{48\gamma + 9a_2^2(\Lambda/\Lambda_1)^2}{16\gamma + 3a_2^2(\Lambda/\Lambda_1)^2}, B = -8\pi G\gamma, C = -\frac{\gamma a_2^2 \Lambda_1}{16}, D = -\frac{3a_2^2 \Lambda}{32}, E = -\frac{4}{a_2}$
BH criteria	(i) $\Lambda_1 > 0$
two horizons	if $\Lambda < 0, \gamma < 0, a_2^2 > \frac{128\Lambda_1^2}{9\Lambda^2} \gamma $
one horizon	if $\gamma > 0$
BH criteria	(ii) $\Lambda_1 < 0$
two horizons	if $\Lambda > 0, \gamma < 0, a_2^2 > \frac{128\Lambda_1^2}{9\Lambda^2} \gamma $
one horizon	if $\gamma > 0$
Curvature	$-\Lambda_1$
Horizon	$\frac{3a_2^2 \Lambda}{32\Lambda_1} \left(-1 \pm \sqrt{1 + \frac{128\gamma\Lambda^2}{9a_2^2\Lambda_1^2}} \right)$
Temperature	$\frac{ \Lambda_1 a_2 }{16\pi} \sqrt{\frac{9a_2^2 \Lambda^2}{64\Lambda_1^2} + 2\gamma}$

Table 3: Characteristics of the solutions of black hole type corresponding to model III.

Characteristics	Model IV
Fields	$g(x) = \frac{A}{x} + Bx, \psi(x) = -4A \left(\frac{C}{x^5} + \frac{E}{x^7} \right), \phi(x) = D \ln x, \omega(\phi) = Ce^{-2\phi/D} + Ee^{-4\phi/D} = \frac{C}{x^2} + \frac{E}{x^4}$
Parameters	$a_1 = \frac{7a_2}{5} = -\frac{7a}{5}, A = \frac{28a_2^2 \Lambda}{775}, B = -\frac{224a_2^4 \Lambda \gamma}{135625}, C = \frac{1550\pi G \gamma}{28a_2^2 \Lambda}, D = -\frac{5}{a_2}, E = -\frac{19375\pi G}{784a_2^4 \Lambda}$
BH criteria	$\gamma > 0, \Lambda < 0$
one horizon	always
Curvature	$-\frac{56a_2^2 \Lambda}{775x}$
Horizon	$\sqrt{\frac{175}{8a_2^2 \gamma}}$
Temperature	$\frac{112a_2^4 \gamma \Lambda }{135625\pi}$

Table 4: Characteristics of the solutions of black hole type corresponding to model IV.

the + sign is chosen. Moreover, we always take $x = |x| = r$, so that x is assumed to be positive.

For the first three models we have found cosmological black hole type solutions, which have been analyzed both for asymptotically de Sitter and anti-de Sitter spacetimes. As we see from the tables, a wide variety of cases with single and double horizons is obtained, depending on the sign of the parameters.

The $\Lambda = 0$ case for models I and II can be obtained straightforwardly by just taking the $\Lambda \rightarrow 0$ limit of all quantities shown on the first two tables, except for the black hole criteria since, in that case, we find one horizon only, located at $x_H = -8\pi Gb/(a^2 B)$, with $\Lambda, b > 0$ and $B < 0$, for the first model, and at $x_H = -\omega_0 a^2 A/(2\pi Gb)$, with $b, \omega_0 > 0$ and $A < 0$, for the second one.

The solution for model IV corresponds to a real black hole with an event horizon. It exhibits a metric and curvature singularity at $x = 0$. Notice that for $\gamma < 0$ this singularity will be a naked one. Other interesting quantities, as the ADM mass (which turns out to be zero or, better, ‘compatible’ with zero), the curvature and the temperature of the black holes are also given, in order to obtain a sufficiently detailed picture of the models considered.

Summing up, we have investigated here the black hole solutions of several models of dilatonic gravity coupled to a Liouville theory. It is interesting to observe that in the case of JT dilatonic gravity (without the field ϕ) a regular black hole of cosmological type has been first discovered in Ref. [14]. Our results provide a straightforward extension of this type of black holes for the case when an additional scalar field is present. In a similar way one can construct black hole solutions for the other versions of (3).

4. The one-loop effective action. Let us now consider the theory (3) at the quantum level. Using the standard Schwinger-De Witt algorithm and the background field method, the corresponding quantum effective action can be calculated without problems. For the case of standard dilatonic gravity this has been done in great detail in Refs. [15, 16]. That calculation can be repeated here, to deal with the more complicated cases in which we are interested. The starting point of the background field method is now

$$\phi_i \longrightarrow \phi_i + \varphi_i, \quad \chi_a \longrightarrow \chi_a + \eta_a, \quad g_{\mu\nu} \longrightarrow e^\sigma g_{\mu\nu}. \quad (9)$$

The calculation is, however, rather tedious and straightforward, since it just makes use of the well-known techniques mentioned already, so we have decided to spare the reader all details and to present the final results only. They correspond to the one-loop effective action in the conformal gauge, and are:

$$\begin{aligned} \Gamma_{div} = & \frac{1}{4\pi\epsilon} \int d^2x \sqrt{-g} \left\{ \left(\frac{23-n}{6} + C_i^i \right) R + V_i^i + 2G_i^* V^i + G^{**} V \right. \\ & + \left(\frac{f_i^i}{2} - \frac{f_i f^i}{2f} \right) g^{\mu\nu} \partial_\mu \chi_a \partial_\nu \chi^a - \left(Z_{(ij)}^i + G^{i*} C_{ij} \right) \Delta \phi^j \\ & \left. + \left[\frac{1}{4} A_{ij}(\phi) + n \frac{f_i f_j}{4f^2} \right] g^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_j \right\}, \end{aligned} \quad (10)$$

where

$$G_{AB} = \begin{pmatrix} Z_{(ij)} & C_j \\ C_i & 0 \end{pmatrix}, \quad G^{AB} = (G_{AB})^{-1} = \begin{pmatrix} G^{ij} & G^{j*} \\ G^{i*} & G^{**} \end{pmatrix}, \quad (11)$$

and G_{AB} is supposed to be invertible. The indices are rised with the spacetime metric corresponding to the inverse configuration, G^{ij} , for instance,

$$V^i = G^{ij}V_j, \quad Z_{(j)}^{(i)} = G^{ik}Z_{(kj)}. \quad (12)$$

The derivatives of the functions in (10) are denoted with lower latin indices, e.g. $C_i = \frac{\delta C}{\delta \phi_i}$ and, in (10),

$$\begin{aligned} \mathcal{A}_{kl}(\phi) = & Z_k^{(ij)}Z_{(ij)l} + 2Z_{(ik)}^jZ_{(jl)}^i - 2Z_{(k)j}^iZ_{(il)}^j + 4G^{i*}G^{j*}C_{ik}C_{jl} \\ & + 2[G^{i*}C_k^j(Z_{(ij)l} + Z_{(jl)i} - Z_{(il)j}) + (k \leftrightarrow l)] \\ & + 2Z_{(kl)i}^i - 2Z_{(ik)l}^i - 2Z_{(il)k}^i - 4G^{i*}C_{ikl}. \end{aligned} \quad (13)$$

The one-loop effective action (10) looks rather complicated, although it shows explicitly that the theory is one-loop renormalizable. By looking at the zeros of the generalized beta functions one should be able to find the fixed points of the generalized renormalization group, and to construct the corresponding finite dilatonic gravity models of type (3). In particular, it follows from (10) that the theory (3) with all dilatonic coupling functions being constants is finite (for $V = 0$, $n = 23$), and such a model is certainly a fixed point of the renormalization group.

Specifying the general answer (11) to the model (2) one can easily obtain the one-loop effective action corresponding to (2), e.g. in the formulation of the theory with the auxiliary field ψ . After eliminating the auxiliary field by means of the constraint $\psi = 2\omega(\phi)R$ —the usual way to proceed in the background field method (see, for example, [17])— we get

$$\begin{aligned} \Gamma_{div} = & \frac{1}{4\pi\epsilon} \int d^2x \sqrt{-g} \left\{ \left(\frac{11-n}{6} + \frac{C''}{Z} - \frac{2C'\omega'}{\omega Z} \right) R - \frac{(1/\omega)''\omega^2}{Z} R^2 + \frac{V''}{Z} - \frac{(C')^2}{2\omega Z} \right. \\ & \left. + \left(\frac{n(f')^2}{4f^2} - \frac{3(Z')^2}{4Z^2} + \frac{Z''}{2Z} \right) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \left(\frac{f''}{2Z} - \frac{(f')^2}{2fZ} \right) g^{\mu\nu} \partial_\mu \chi_i \partial_\nu \chi^i \right\}, \end{aligned} \quad (14)$$

The R^2 -term in (14) can be eliminated via the equation of motion $\delta S/\delta\phi = 0$ for the theory (1).

Starting from (1) a whole set of models that are one-loop finite can be given, which hence correspond to the fixed points of the generalized renormalization group. One of this examples is given by model (1) itself with the following dilatonic functions:

$$\begin{aligned} n = 11, \quad Z(\phi) &= \frac{\alpha_3 \exp(\alpha_2 \phi)}{[\alpha_4 \exp(2\alpha_2 \phi) + 1]^2}, \quad f(\phi) = \alpha_1 \exp(\alpha_2 \phi / \sqrt{11}), \\ \omega(\phi) &= \text{const.}, \quad C(\phi) = 2\alpha_5 \phi + \alpha_6, \quad V(\phi) = \frac{\alpha_5^2}{\omega} \phi^2 + \alpha_7 \phi + \alpha_8, \end{aligned} \quad (15)$$

where $\alpha_1, \dots, \alpha_8$, are arbitrary constants.

It is also interesting to remark that—as in the case of standard dilatonic gravity (see [18] and the paper by Chamseddine in [12])—the model (3) can be easily represented, in the conformal gauge $g_{\mu\nu} = e^\sigma \bar{g}_{\mu\nu}$, as a sigma model of the special form:

$$S = \int d^2x \sqrt{-\bar{g}} \left(\frac{1}{2} G_{\tilde{A}\tilde{B}} \bar{g}^{\mu\nu} \partial_\mu \tilde{\phi}^{\tilde{A}} \partial_\nu \tilde{\phi}^{\tilde{B}} + \bar{R}\psi + T \right), \quad (16)$$

with $\tilde{\phi}^{\tilde{A}} = (\phi_i, \sigma, \chi_a)$, and where

$$G_{\tilde{A}\tilde{B}} = \begin{pmatrix} Z_{(ij)} & C_j & 0 \\ C_i & 0 & 0 \\ 0 & 0 & f(\phi)\delta_{ab} \end{pmatrix}, \quad \psi = C(\phi), \quad T = e^\sigma V(\phi). \quad (17)$$

Then an alternative way to calculate the one-loop effective action (13) is to use the standard string β -functions of the σ -model approach [19, 20], with the background (17). An interesting (although not easy) task would be to try to understand the connection between the conformal properties of the σ -model (16) and those of the black hole type solutions that appear in the related theory of string-inspired gravity (similar to Ref. [3]). We plan to return to these questions in the near future.

In summary, we have studied in this letter a quite general (one-loop) renormalizable theory of dilatonic gravity coupled to a scalar field. For some particular cases of this theory, we have investigated in some detail their cosmological black hole solutions, obtaining their main characteristics, as the ADM mass, the horizon structure and the black-hole temperature. To conclude, the one-loop effective action has been obtained, and the precise connection of the theory with its formulation in terms of a sigma model has been described.

Acknowledgments

SDO would like to thank A. Chamseddine for discussions. This work has been supported by DGICYT (Spain), project Nos. PB93-0035 and SAB93-0024, by CIRIT (Generalitat de Catalunya), and by RFFR (Russia), project No. 94-02-03234.

References

- [1] M.B. Green, J.H. Schwarz and E. Witten, *Superstring theory* (Cambridge Univ. Press, Cambridge, 1987).
- [2] L. Álvarez-Gaumé, D.Z. Freedman and S. Mukhi, Ann. Phys. (NY) **134** (1981) 85; D. Freedman, Ann. Phys. (NY) **163** (1985) 318; H. Osborn, Ann. Phys. (NY) **200** (1990) 1.
- [3] E. Witten, Phys. Rev. **D44** (1991) 314.
- [4] I. Antoniadis, J. Rizos and K. Tamvakis, Nucl. Phys. **B415** (1994) 497.
- [5] M. McGuigan, C. Nappi and S. Yost, Nucl. Phys. **B375** (1992) 421; E. Elizalde and S.D. Odintsov, Nucl. Phys. **B399** (1993) 581.
- [6] R.B. Mann, Phys. Rev. **D47** (1993) 4438; Nucl. Phys. **B418** (1994) 231.
- [7] E. Elizalde, S. Naftulin and S.D. Odintsov, Phys. Lett. **B323** (1994) 124.
- [8] I.L. Buchbinder, V.A. Kryhtin and V.D. Pershin, preprint TSPI-TH1/94 (1994).
- [9] I.L. Buchbinder, S.D. Odintsov and I.L. Shapiro, *Effective action in quantum gravity* (IOP, Bristol and Philadelphia, 1992).
- [10] T. Yoneya, Phys. Lett. **B149** (1984) 111.
- [11] I. Jack and D.R.T. Jones, Nucl. Phys. **B358** (1991) 695.
- [12] R. Jackiw, in *Quantum theory of gravity*, Ed. S. Christensen (Hilger, Bristol, 1984); C. Teitelboim, Phys. Lett. **B126** (1983) 41; A.H. Chamseddine, Nucl. Phys. **B368** (1992) 98; I.M. Lichtzier and S.D. Odintsov, Mod. Phys. Lett. **A6** (1991) 1953; D. Cangemi and R. Jackiw, Ann. Phys. (NY) **225** (1993) 229.
- [13] K. Chan and R.B. Mann, preprint WATPHYS TH-94/10 (1994).
- [14] D. Christensen and R.B. Mann, Class. Quant. Grav. **9** (1992) 1769; R.B. Mann, A. Sheikh and L. Tarasov, Nucl. Phys. **B341** (1990) 134; A. Achúcarro and M.E. Ortiz, Phys. Rev. **D48** (1993) 3600; J.P.S. Lemos and P. Sa, Class. Quant. Grav. **11** (1994) L11.
- [15] S.D. Odintsov and I.L. Shapiro, Class. Quant. Grav. **8** (1991) L57; Phys. Lett. **B263** (1991) 183; Int. J. Mod. Phys. **D1** (1993) 571.
- [16] R. Kantowski and C. Marzban, Phys. Rev. **D46** (1992) 259.
- [17] P. West, *Introduction to supersymmetry and supergravity* (World Sci., Singapore, 1986), Chap. 17.
- [18] J. Russo and A. Tseytlin, Nucl. Phys. **B382** (1992) 259.
- [19] E.S. Fradkin and A.A. Tseytlin, Nucl. Phys. **B261** (1985) 1.
- [20] C. Callan, D. Friedan, E.J. Martinec and M. Perry, Nucl. Phys. **B262** (1985) 593.